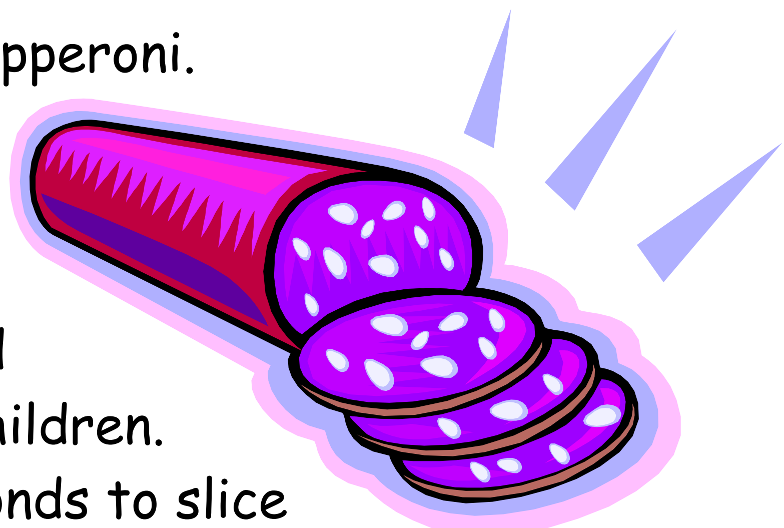
2009 REGIONALS Grade 7

PROBLEM ONE



David has sticks of pepperoni. One stick has to be shared between four adults and the other stick has to be shared between six hungry children. It takes him nine seconds to slice the first stick into 4 round pieces.

HOW MANY SECONDS WILL IT TAKE DAVID TO SLICE THE CHILDREN'S STICK OF PEPPERONI INTO 6 ROUND PIECES?

2009 REGIONALS Grade 7

PROBLEM TWO

A pizza delivery man dropped a pizza off at Emily's party at 6 pm. The pizza was a giant square shape and had been sliced into 36 pieces. Emily's brother had to go pick up some movies so he asked Emily to make sure that there was some pizza left over for him. By 6:30 pm, $\frac{8}{12}$ of the pizza had been eaten. By 7 pm $\frac{1}{3}$ of the remaining pizza was eaten, and by 7:30 pm $\frac{3}{4}$ of the remaining pizza was eaten.

HOW MANY PIECES OF PIZZA ARE LEFT OVER FOR EMILY'S BROTHER TO EAT?

2009 REGIONALS Grade 7

PROBLEM THREE

Erica is learning how to skate and wants a pair of new skates. Erica's mother does not feel she is quite ready for a new pair of skates but said that once she practices for another two hours, she would buy her a pair. Erica practices for 8 minutes the first day, 16 minutes the second day, 24 minutes the third day, and so on.

HOW MANY DAYS DOES IT TAKE ERICA TO GET HER NEW SKATES?

2009 REGIONALS Grade 7

PROBLEM FOUR

There are two sizes of tables at the hall where your cousin's wedding reception will be held. One size table seats five people, and the other seats eight people. Seventy-nine people will be at the wedding reception. The total number of tables that will be set must be less than one dozen. Your cousin needs to know how many tables of each size to set and she does not want there to be any empty seats. She asks you for help because of your math expertise.

HOW MANY TABLES OF FIVE NEED TO BE SET?

2009 REGIONALS Grade 7

PROBLEM FIVE

Misty Mouse's bag of treats contains 400 candies, some red, some green, and some pink, Misty's favourite colour. Misty wanted to know how many of her candies are pink. She grabbed a random sample and counted 8 red candies, 6 pink candies, and 2 green candies.

BASED ON MISTY'S RANDOM SAMPLE, HOW MANY OF THE CANDIES IN HER BAG CAN SHE EXPECT TO BE PINK?

Solutions REGIONALS GRADE SEVEN
PROBLEM ONE

NOT AVAILABLE

Solutions REGIONALS GRADE SEVEN
PROBLEM TWO

Solution 1:

$36 \times \frac{8}{12} = \frac{288}{12} = 24$ $\quad$ 36-24=12

$12 \times \frac{1}{3} = \frac{12}{3} = 4$ $\quad$ 12-4 = 8

$8 \times \frac{3}{4} = \frac{24}{4} = 6$ $\quad$ 8-6= 2

<u>2 pieces remain</u>

Solution 2:

6:30pm $36 - (36 \times \frac{8}{12}) = 12$

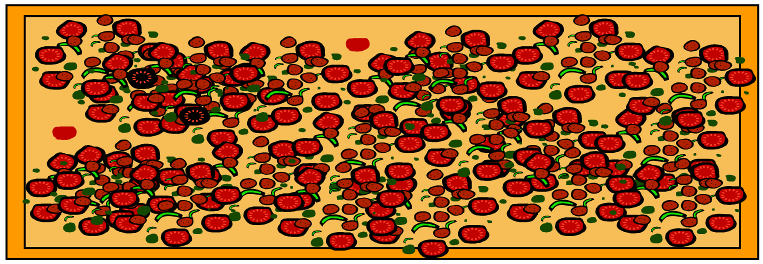
7:00pm $12 - (12 \times \frac{1}{3}) = 8$

7:30pm $8 - (8 \times \frac{3}{4}) = 2$

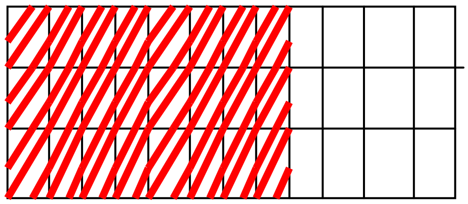
<u>2 pieces remain</u>

Solution 3:

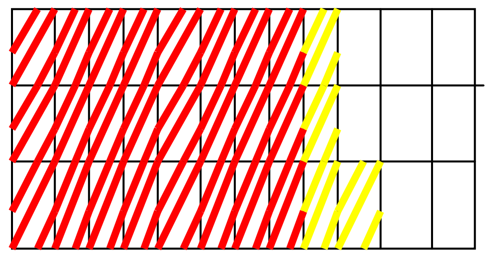
(Thinking of it as a pizza)



The rectangle would be divided first into 36 pieces then shade in. $\frac{8}{12}$

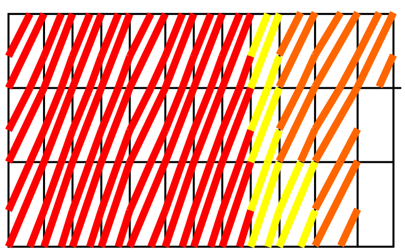


Then divide the remaining 12 pieces by $\frac{1}{3}$.



Then divide the remaining 8 pieces by $\frac{3}{4}$.

Two pieces remain



Solutions ZONE GRADE SEVEN PROBLEM THREE

Method 1: Find a pattern.
Looking at the sequence 8, 16, 24…, one can see that that difference between each number is 8. So continuing this pattern, the sequence will be 8, 16, 24, 24+8=32, 32+8=40, 40+8=48 …

2 hours = 120 minutes. Add up the numbers in the sequence until the sum equals 120.
8 + 16 + 24 +32 + 40 = 120. There are 5 numbers in the sequence; therefore it takes 5 days for Erica to get her new skates.

Method 2: Elimination
The sequence is increasing by multiples of 8, and it needs to add to 120 minutes. One can work with a top down approach. Start with a high multiple of 8 (i.e. 56), then add on the one lower than that, 48, then the one lower, 40, one lower again, 32, etc. until the summation is over 120 minutes. Once the summation is over 120 minutes, one can deduce that the starting number is over the last number in the sequence. That number can then be removed. Continue using this process with the previous number in the sequence (i.e. 48) until your answer sums to 120. The number that is left at the start is the highest amount of time Erica will practice, which is 40. After counting the terms in the sequence, it takes Erica 5 days to complete 2 hours of practicing.

Method 3: Partial Sums
There is a pattern in the partial sums, that is, when adding the previous time onto the next time in the sequence. The first entry in the sequence is 8, which is 8 x 1. The second entry in the sequence is 8 + 16 = 24, which is equal to 8 x 3. The third entry is 24 + 24 = 48, which is equal to 8 x 6 .The forth is 48 + 32 which is equal to 8x10, and the fifth is 80 + 40 which is equal to 8 x 15. Each sum is a multiple of 8 and increased by an extra 8 each time. The difference between 8 x 1 and 8 x3 is 8 x 2. The difference between 8 x 3 and 8 x 6 is 8 x 3. The difference between 8 x 6 and 8 x 10 is 8 x 4 and the difference between 8 x 10 and 8 x 15 is 8 x 5. It increases by a multiple of 8 each time. Therefore, you can begin with this pattern until you reach an entry of 8 x 15 which equals 120 minutes, or two hours. The partial sums can also be shown in a table:

Partial sums	Difference in partial sums
8 x1	
	8x 2
8 x3	
	8x3
8x6	
	8 x4
8x10	

It is clear that the difference is always increasing by 1 multiple of 8 (as shown in column 2), and the next difference in partial sums will be 8 x5. Adding 8 x 5 to the last partial sums (8 x 10) will give you the 8 x 15, or 120 minutes.

Solutions ZONE GRADE SEVEN PROBLEM FOUR

Method 1: Logical Reasoning

Looking at 5 and 8, neither have a multiple with the last digit 9 under the value of 79. Therefore having only tables of 5 or only tables of 8 is not possible. There must be a combination of the two sizes. Since there are no multiples of 5 and 8 that have the last digit 9, there must also be an odd number of tables of 5; otherwise, the multiple will have a 0 in the ones place and there would be no way to end up with exactly 79 people sitting at the tables. Because there must be an odd number of tables of 5, the number of people sitting at tables of 5 must end with the digit of 5. This means that the number of people sitting at tables of 8 must end in 4 to obtain the second digit 9 in 79. Going through the multiples of 8, there is 24 and 64 that end in 4. By observation, 24 is too small of a number therefore there must be 64 people in tables of 8. This would imply that there are 8 tables of 8. This leaves 15 people, and you can sit those 15 people in 3 tables of 5 without any empty seats.

Method 2: Algebra

Let x = the number of tables of 8
Let y = the number of tables of 5
$8x + 5y = 79$
$5y = 79 - 8x$
At this point, y must be a whole number, so 79 – 8x must be a multiple of 5. Plugging in values less than 12 for x and y, the only pair of variables that work are y =3 and x= 8.

Method 3: Table of values

One can make a table of values to solve this problem by entering various numbers into a table. One column has the number of tables of 8, one has the number of tables of 5, one has the total number of tables and the last column has the number of people this combination of tables accommodates. The total number of tables must be less than 12. Coming up with various combinations of numbers that add to give less than 12, one can calculate the number of people each combination of tables accommodates. This method will continue until 79 is reached.

Tables of 8	Tables of 5	Total Tables	Total # of People
11	0	11	88
10	1	11	85
10	0	10	80
9	2	11	82
9	1	10	77
8	3	11	79

Solutions ZONE GRADE SEVEN PROBLEM FIVE

Solution 1:

The random sample was of 16 so the probability of each was:

8 Reds = $\frac{8}{16} = \frac{1}{2}$

6 pink = $\frac{6}{16} = \frac{3}{8}$

2 green = $\frac{2}{16} = \frac{1}{8}$

Probability of pink = $\frac{3}{8}$

400 × $\frac{3}{8} = \frac{1200}{8}$ = 150

There should be approximately 150 pink candies in the bag.

Solution 2:
The random sample was of 16 so the probability of each was:

8 Reds = $\frac{8}{16}$ = $\frac{1}{2}$

6 pink = $\frac{6}{16}$ = $\frac{3}{8}$

2 green = $\frac{2}{16}$ = $\frac{1}{8}$

Probability (P)
Total # of candies = P(green) + P(red) + P(pink)
400 = **P(pink) + (400*** $\frac{1}{2}$) + (400* $\frac{1}{8}$)
400 = **P(pink) + 200 + 50**
400 = **P(pink) + 250**
400 - 250= **P(pink)**
P(pink) = 150
Approximately 150 pink are in the bag.

Solution 3:
The random sample was of 16 so the probability of the pink is:

6 Pink = $\frac{6}{16}$ = $\frac{3}{8}$ =37.5%

So in a bag of 400 , we can expect:
400 x 37.5%= 150
There should be 150 pink candies in the bag.

2009 REGIONALS
GRADE SEVEN
ANSWER KEY

PROBLEM ONE - 15

PROBLEM TWO - 2

PROBLEM THREE - 5

PROBLEM FOUR - 3

PROBLEM FIVE - 150

2009 REGIONALS

Grade 8

PROBLEM ONE

Aaron , a student teacher, created a science quiz that had 96 questions. Seventy-five percent of the quiz was about water. He would like to fix the quiz so that ninety percent of the questions are about water. After deleting a number of non-water questions, Aaron succeeds in making ninety percent of the quiz questions about water.

HOW MANY QUESTIONS IN TOTAL ARE ON THE NEW QUIZ?

2009 REGIONALS Grade 8

PROBLEM TWO

Two painters are painting a house using two ladders. Both ladders are 17 meters in length and are leaning against a wall of the house. The foot of one of the ladders is 15 meters away from the base of the house and the other ladder is 8 meters away from the same base of the house.

WHAT IS THE DIFFERENCE IN HEIGHT OF THE TOPS OF THE LADDERS?

2009 REGIONALS Grade 8

PROBLEM THREE

Indiana Bones is on a new adventure and has followed the clues to the entrance of the temple of the ancient Fractionista tribe. In order to enter the sacred temple, he must solve a problem involving their beloved fractions. His sidekick, Mutt, doesn't think it's possible to solve, but Indy hasn't given up…

This is the problem:

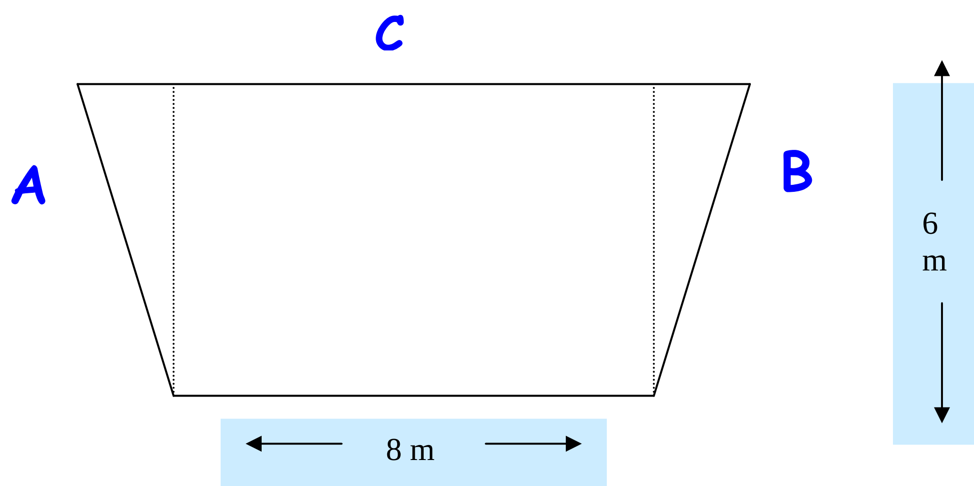
$$\frac{2}{\frac{1}{2}} + \frac{4}{\frac{1}{4}} + \frac{6}{\frac{1}{6}} + \frac{8}{\frac{1}{8}} \;\ldots\; \frac{16}{\frac{1}{16}} =$$

WHAT IS THE SOLUTION THAT WILL ALLOW THEM TO ENTER THE TEMPLE?

2009 REGIONALS Grade 8

PROBLEM FOUR

Meg, Jess, and Sam are moving into a new, but smaller, apartment. They decide to sell some of their belongings at the local flea market.



They rent a $54m^2$ spot for $108. Each one will pay a fee according to how much space they use. Jess takes the largest space, C. Meg and Sam take spaces A and B, which are the same size. When it was time to pay for the space, Jess did not have enough money with her and so she borrowed some money from Meg. Sam paid *exactly* what she should have and Meg paid 3 times *more* than she should have. Meg is anxious for Jess to pay her back.

HOW MUCH MONEY DOES JESS OWE MEG?

2009 REGIONALS Grade 8

PROBLEM FIVE

The Casual family are on a road trip in a rental car. Papa Casual wants to return the car with its 60-litre gas tank completely empty, since they prepaid for a full tank of gas and won't get a refund for any unused gas. They have driven two-thirds of the distance on a 900-km highway trip and the tank is now one quarter full. They pull into the last gas station they will pass before returning to the car rental place.

CHOOSE THE ANSWER BELOW THAT SHOWS EXACTLY HOW MANY LITRES OF GAS PAPA CASUAL SHOULD PUT IN THE TANK.

$22\frac{1}{2}$	$7\frac{1}{2}$	22	7	15

Solutions
REGIONALS GRADE EIGHT
PROBLEM ONE

Method 1

Represent percents as fractions. 75% = $\frac{3}{4}$ and $\frac{3}{4}$ of 96 is 72.

Also, 90% = $\frac{9}{10}$ and if $\frac{9}{10}$ of a number is 72, then $\frac{1}{10}$ of the number is 8 and $\frac{10}{10}$ of the number is 80. The test had to have only 80 questions.

Method 2

Use algebra and decimals. If N is the number to be found, then 75% of 96 is 90% of N

$0.75 \times 96 = 0.90N \rightarrow 72 = 0.90N \rightarrow N = 72 \div 0.90 = 80$

Method 3

Use algebra and proportions. Let N be the number to be found and x be 75% of 96.

(1) $\frac{75}{100} = \frac{x}{96} \rightarrow \frac{3}{4} = \frac{x}{96} \rightarrow 4x = 288 \rightarrow x = 72$ (sub into equation 2)

(2) $\frac{90}{100} = \frac{72}{N} \rightarrow \frac{9}{10} = \frac{72}{N} \rightarrow 9N = 720 \rightarrow N = 80$.

<u>REGIONALS GRADE EIGHT   PROBLEM TWO</u>

Solution 1:

The ladder whose base is closer to the wall would have a greater slope because of the greater rise. Therefore the second ladder has the greater slope. Using Pythagorean Theorem to find out how high up the wall the ladder will go:

$A^2 + B^2 = C^2$

82 + x^2 = 172

64 + x^2 = 289

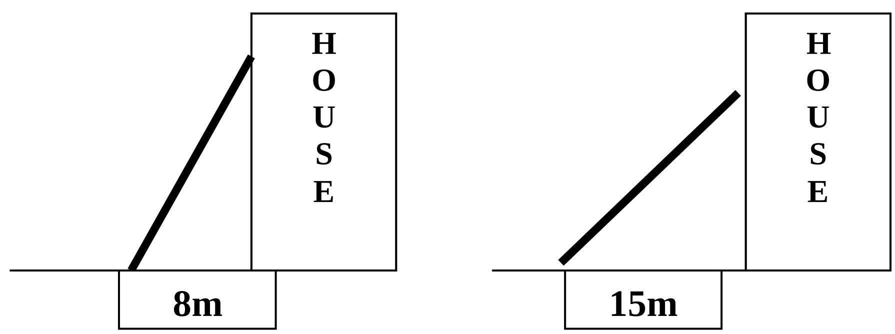
$x^2 = 289 - 64$

$x^2 = 225$

$x = 15$

The ladder goes 15m up the wall. The same procedure reveals that the other ladder is 8m up the wall. The difference in height of where the tops of the ladders are is 7.

Solutions 2: Draw a picture to see the different ladder slopes.



The ladder that is 8 meters away from the wall has the greater slope therefore Pythagorean Theorem would be used to find the height of the wall the ladder reaches.

$A^2 + B^2 = C^2$ (where C=17m and A=8m)

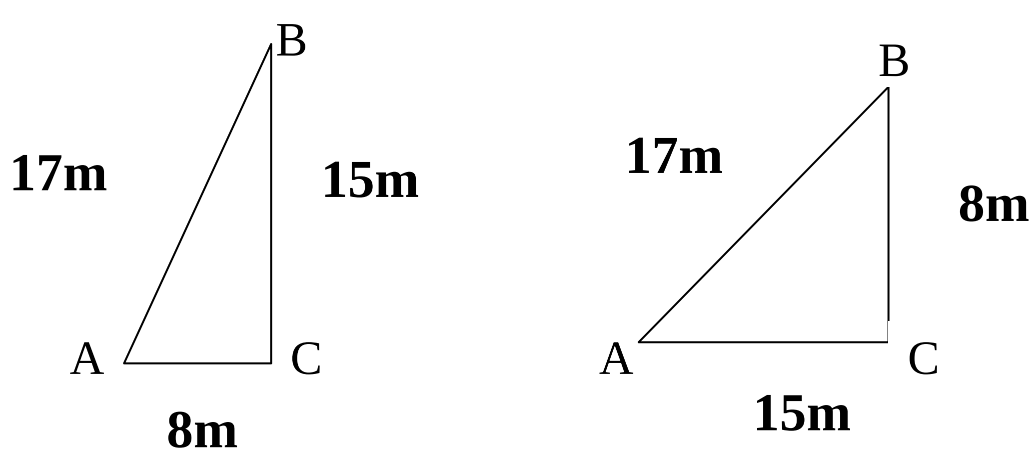
$8^2 + A^2 = 17^2$

$64 + A^2 = 289$

$A^2 = 289 - 64$

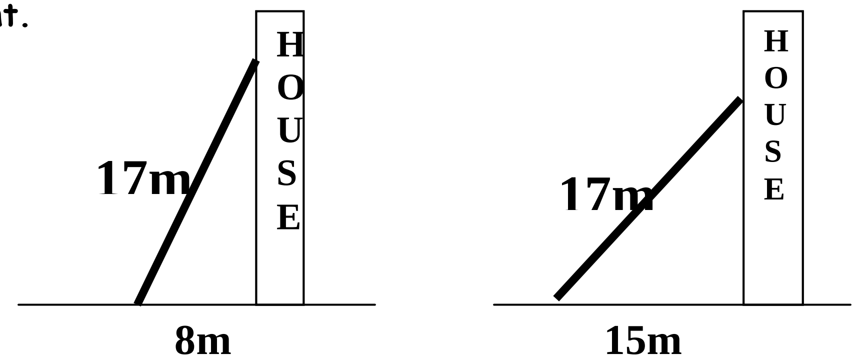
$A^2 = 225$

A = 15m



The ladder reaches 15 meters high on the wall of the house. The same procedure reveals that the other ladder is 8m up the wall. The difference in height of where the tops of the ladders are is 7.

Solution 3:
Solve it as two triangle using Pythagorean Theorem then find the difference in height.



$C^2 = A^2 + B^2$
$17^2 = A^2 + 8^2$
$289 = A^2 + 64$
$289 – 64 = A^2$
$225 = A^2$
$A = 15$

$C^2 = A^2 + B^2$
$17^2 = A^2 + 15^2$
$289 = A^2 + 225$
$289 – 225 = A^2$
$64 = A^2$
$A = 8$

Triangle 1 reaches a height 15 meters. Triangle 2 reaches a height of 8 meters. The difference in height is 7 meters.

Solutions
REGIONALS GRADE EIGHT
PROBLEM THREE

***Answer*:** 816

$$2^2+4^2+6^2+8^2+10^2+12^2+14^2+16^2=$$

Solutions
REGIONALS GRADE EIGHT
PROBLEM FOUR

Method A

Find the area that each individual used in order to calculate how much each person should pay. This can be done by using the formula for finding the area of a trapezoid: A = ½ h(a+b); *where h is the height and a and b are the 2 sides perpendicular to the height*

A = ½ h(a+b)
54 = ½ $\times$6(a + 8)
54 = 3(a +8)
54 = 3a + 24
30 = 3a
a = 10

Calculate the area of both Meg and Sam's space using the formula for the area of a triangle: A = ½ bh

A = ½ (1$\times$6)
A = 3m^2

The area used by Jess can be calculated using the formula for finding the area of a rectangle:

A = lw; *where l is the length and w is the width*

A = 8 $\times$6
A = 48m^2

The cost per metre is \$108$\div$54 = \$2/m^2
The cost paid by Meg: 3 (\$2 $\times$3m^2) = 3(\$6) = \$18
The cost paid by Sam: \$2 $\times$3m^2 = \$6
\$18 + \$6 = \$24 and \$108 - \$24 = \$84, what Jess actually paid.
Since the amount that Jess should pay can be calculated by the following:
48m^2 $\times$2 = \$96; and
\$96 - \$84 = \$12 is the amount that she owes Meg.

Method B
Find the area that each individual used in order to calculate how much each person should pay. This can be done by using the formula for finding the area of a rectangle and 2 triangles:
$A = lw + \frac{1}{2} bh + \frac{1}{2} bh$
$54 = 6\times8 + bh$
$54 = 48 + b\times6$
$6 = 6b$
$b = 1$
Calculate the area of both Meg and Sam's space using the formula for the area of a triangle: $A = \frac{1}{2} bh$
$A = \frac{1}{2} (1\times6)$
$A = 3m^2$
The area used by Jess can be calculated using the formula for finding the area of a rectangle:
$A = lw$; *where l is the length and w is the width*
$A = 8 \times6$
$A = 48m^2$
The cost per metre is $\$108\div54 = \$2/m^2$
The cost paid by Meg: $3 (\$2 \times3m^2) = 3(\$6) = \$18$
The cost paid by Sam: $\$2 \times3m^2 = \$6$
$18 + $6 = $24 and $108 - $24 = $84, what Jess actually paid.
Since the amount that Jess should pay can be calculated by the following:
$48m^2 \times2 = \$96$; and
$96 - $84 = $12 is the amount that she owes Meg.

Method C
Use the area of a rectangle, A = lw, *where l is the length and w is the width* to calculate what Jess should pay, and use the information given in the problem to see what she actually did pay and how much she owes:
$A = 6 \times8$
$A = 48m^2$ is the area that Jess used
The cost per metre is $\$108\div54 = \$2/m^2$
$48m^2 \times2 = \$96$ is the amount that Jess should pay for the space

The total amount paid is $108, so $108 – $96 = $12, the amount the other 2 should pay.
Since we know that they split the space equally, they should each pay $\$12 \div2 = \$6$, but we know that only Sam paid $6; Meg paid $3m^2 \times \$6 = \$18$.

$\$6 + \$18 = \$24$, the total amount paid by Sam & Meg. $\$108 - \$24 = \$84$, what Jess actually paid.

Since Jess should have paid \$96 for her share of the space but only paid \$84, she owes Meg \$12 ($\$96 - \$84 = \$12$).

Solutions
REGIONALS GRADE EIGHT
PROBLEM FIVE

Method 1:
Set up a comparison of proportions to figure out how many Litres of gas it takes to go 1 km.

Then multiply this answer by 300, which is the number of kilometers left to go:

Since there are still 15 Litres in the tank (1/4 x 60 Litres), Bart and Homer would need 22.5-15 = 7.5 Litres.

Method 2:
Solve algebraically by setting their variable = to either the number of kilometers the car will go per 1 Litre of gas.

The number of Litres needed to complete the entire trip would be:

Since 60 Litres have already been put in the tank, 67.5-60=7.5 Litres are needed.

Method 3:
Set up a comparison of two ratios.

Solve for x:
(45)(900) = 600x
x=40 500/600
x=67.5 Litres

This would tell them the number of liters needed for the entire trip to be 67.5. Since 60 Litres (a full tank) were already put in, an additional 7.5 Litres will need to be added. 67.5-60 = 7.5 Litres.

2009 REGIONALS
GRADE EIGHT
ANSWER KEY

PROBLEM ONE - 80

PROBLEM TWO - 7

PROBLEM THREE - 816

PROBLEM FOUR - 12

PROBLEM FIVE - $7 \frac{1}{2}$

2009 REGIONALS Grade 9

PROBLEM ONE

Triangle ABC has sides 10, 24, and 26 cm long.

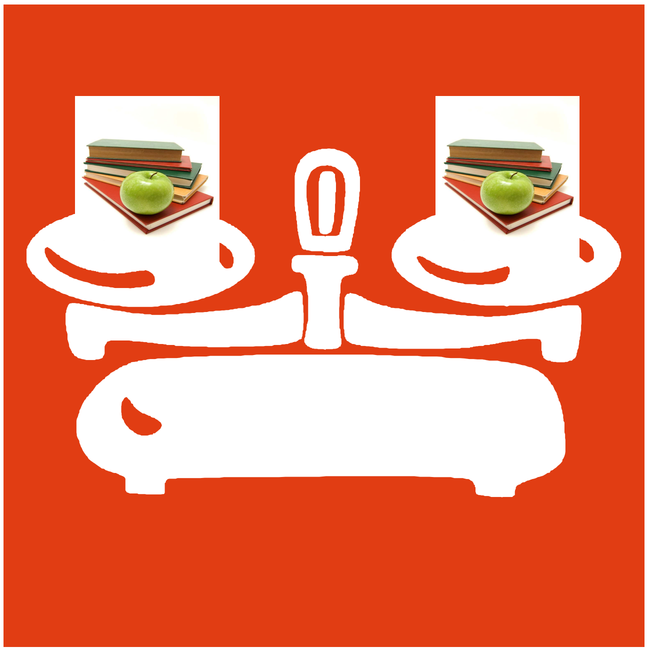
Rectangle DEFG has an area equal to that of the triangle, and is 3 cm wide.

WHAT IS THE PERIMETER OF THE RECTANGLE?

Hint: What type of triangle is triangle ABC?

2009 REGIONALS Grade 9

PROBLEM TWO



Kim has finally found a good use for textbooks. She is using them to test the accuracy of a new set of giant scales at her school.

What she has determined so far is that:

- 8 Science books weigh as much as 15 English books
- 9 Math books weigh as much as 4 Science books
- 10 English books weigh as much as 27 French books.

HOW MANY FRENCH BOOKS WOULD SHE NEED TO EQUAL E WEIGHT OF FOUR MATH BOOKS?

2009 REGIONALS Grade 9

PROBLEM THREE

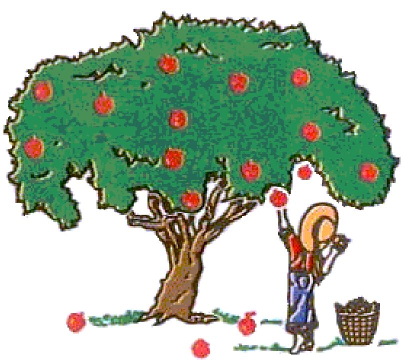
Brian loves bananas. In fact, he loves them so much that he decided to only eat bananas for five days. He started his banana diet on a Monday. Every day he ate 6 more bananas than the previous day. By the end of the day on Friday, Brian had consumed 100 bananas!

HOW MANY BANANAS DID BRIAN EAT ON WEDNESDAY?

2009 REGIONALS Grade 9

PROBLEM FOUR

Every fall, Lola helps her grandparents carry apples from the orchard to their house. Lola has to move a total of 300kg of apples. She has to choose between using either **two buckets** or **one wheelbarrow** to do this job. Each bucket holds 750g of apples, while the wheelbarrow can hold 2000g. However, a round-trip with the buckets is 0.25 km, while a round-trip with the wheelbarrow is twice that distance. Both the buckets and the wheelbarrow are stored at her grandparents' house.

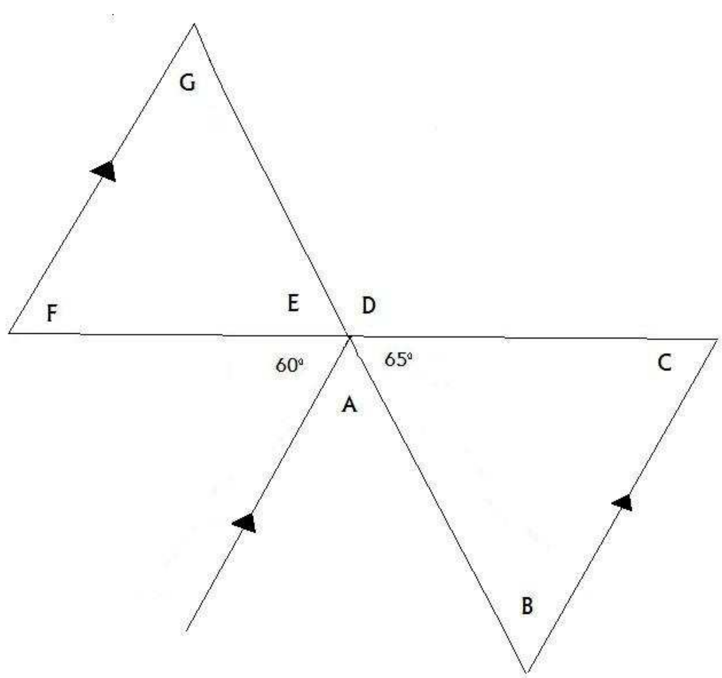


WHAT IS THE SHORTEST DISTANCE, IN KILOMETRES, LOLA HAS TO TRAVEL IN ORDER TO MOVE ALL 300KG OF APPLES?

2009 REGIONALS Grade 9

PROBLEM FIVE

Ranger Radical is coming to PI National Park to do a safety inspection. He is particularly interested in ensuring that all trails are marked with the necessary warning signs. One of the signs he is looking for is the "Warning: Sharp Turn" sign. The rules states that a "Warning: Sharp Turn" sign must be placed at the vertex of two paths which intersect at an angle less than 60º. All paths which intersect at angles greater than or equal to 60º do not require warning signs. PI National Park does not have any "Warning: Sharp Turn" signs.



Lines indicate paths and all paths with arrows are parallel

HOW MANY SIGNS WILL THE PARK NEED TO PUT UP IN ORDER TO PASS INSPECTION?

Solutions REGIONALS GRADE NINE
PROBLEM ONE
NOT AVAILABLE
Solutions REGIONALS GRADE NINE
PROBLEM TWO
NOT AVAILABLE
Solutions REGIONALS GRADE NINE
PROBLEM THREE
NOT AVAILABLE
Solutions REGIONALS GRADE NINE
PROBLEM FOUR
NOT AVAILABLE
Solutions REGIONALS GRADE NINE
PROBLEM FIVE

Method 1: *Use alternate interior angle properties Solve for E, F and C first.*

Solve for E:

E = 65^o since E is vertically opposite to an angle of 65^o

Solve for F and C:

C=60^o since C and 60^o angle given are corresponding angles

F= 60^o since C and F are alternate interior angles

Solve for G and B: Both G and B are the remaining unknown angle within a triangle.

$180^o-60^o-65^o = 55^o$

G=B=55^o

Solve for A:

A = 55^o since A and G are corresponding angles.

A = 55^o since $180^o-60^o-65^o = 55^o$ (A is supplementary to the combination of the 2 given angles)

A, B, and G are the only angles less than 60^o; therefore, the park needs 3 "Warning: Sharp Turn" signs.

Method 2: *Use properties of non-parallel lines and their transversals. Solve for A first.*

Solve for A: The given angles and angle A must add up to 180^o. $180^o- 60^o-65^o = 55^o$

Solve for G and B:

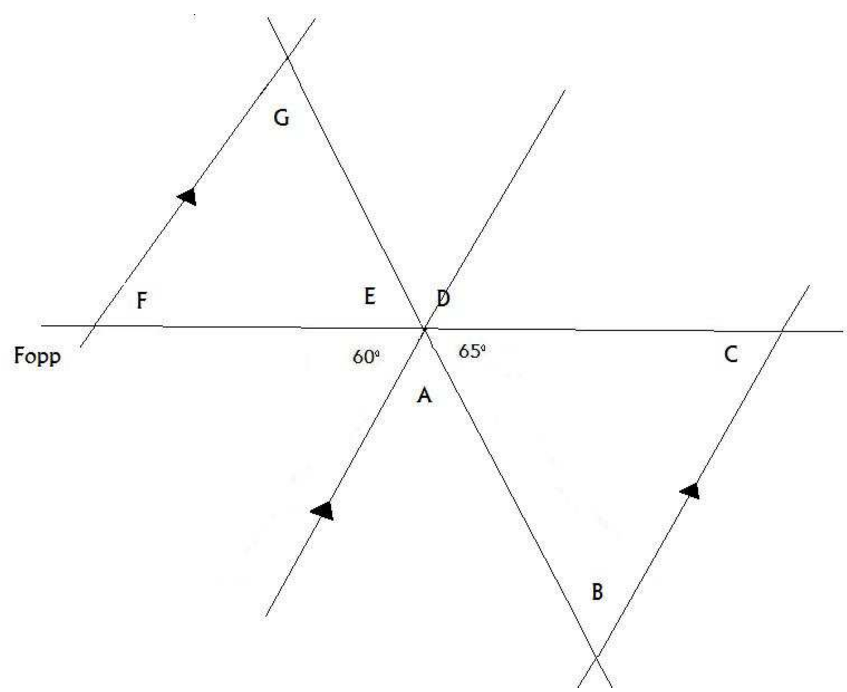
G= 55^o since A and G are corresponding angles.

B= 55^o since the G and B are alternate interior angles

Solve for E:
E = 65^{o} since E is vertically opposite to an angle of 65^{o}
Solve for F and C:Both F and C are the remaining unknown angles within a triangle.
180^{o}-65^{o}-55^{o} = 60^{o}____F=C=60^{o}
A, B, and G are the only angles less than 60^{o}; therefore, the park needs 3 "Warning: Sharp Turn" signs.
Solve for E: E = 65^{o} since E is vertically opposite to an angle of 65^{o}
Solve for F and C:
Fopp = C=60^{o} since C, F_{opp} and 60^{o} angle given are corresponding angles
F= 60^{o} since F_{opp}=60^{o} is vertically opposite angle F
Solve for G and B:Both G and B are the remaining unknown angle within a triangle. 180^{o}-60^{o}-65^{o} = 55^{o}
G=B=55^{o}
Solve for A: A = 55^{o} since A and G are corresponding angles.
A, B, and G are the only angles less than 60^{o}; therefore, the park needs 3 "Warning: Sharp Turn" signs.

Method 3: *Use properties of non-parallel lines and their transversals. Solve for E, F, and C first.*
Solve for E:
E = 65^{o} since E is vertically opposite to an angle of 65^{o}
Solve for F and C:
Fopp = C=60^{o} since C, F_{opp} and 60^{o} angle given are corresponding angles
F= 60^{o} since F_{opp}=60^{o} is vertically opposite angle F

Solve for G and B: Both G and B are the remaining unknown angle within a triangle. 180^{o}-60^{o}-65^{o} = 55^{o}____G=B=55^{o}



Solve for A: A = 55^{o} since A and G are corresponding angles.
A, B, and G are the only angles less than 60^{o}; therefore, the park needs 3 "Warning: Sharp Turn" signs.

Method 4: *Use properties of non-parallel lines and their transversals. Solve for A first.*
Solve for A: The given angles and angle A must add up to 180^{o}. 180^{o}- 60^{o}-65^{o} = 55^{o}
Solve for G and C:
G= 55^{o} since A and G are corresponding angles.

$C = 60^o$ since the 60^o angle and C are corresponding angles.
<u>Solve for E:</u> $E = 65^o$ since E is vertically opposite to an angle of 65^o
<u>Solve for F and B:</u> Both F and B are the remaining unknown angle within a triangle.
$180^o - 60^o - 65^o = 55^o$ $B=55^o$
$180^o - 65^o - 55^o = 60^o$ $F= 60^o$
A, B, and G are the only angles less than 60^o; therefore, the park needs 3 "Warning: Sharp Turn" signs.

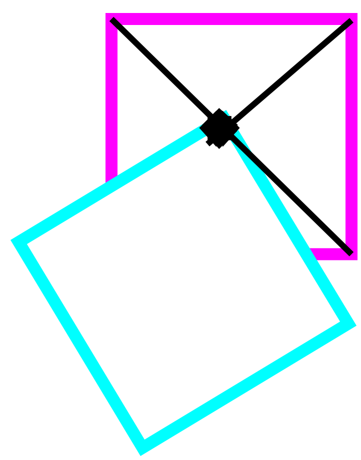
2009 REGIONALS
GRADE NINE
ANSWER KEY

PROBLEM ONE -	86
PROBLEM TWO -	9
PROBLEM THREE -	20
PROBLEM FOUR -	50
PROBLEM FIVE -	3

2009 REGIONAL CHALLENGE

PROBLEM ONE

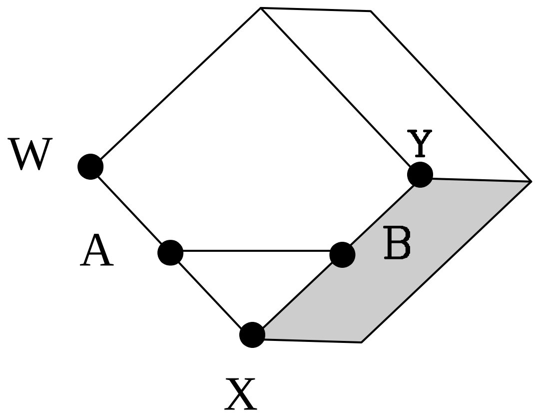
Two identical squares with side lengths of 10 overlap. The corner of one square is fastened at the intersection of the diagonals of the other square and can rotate.

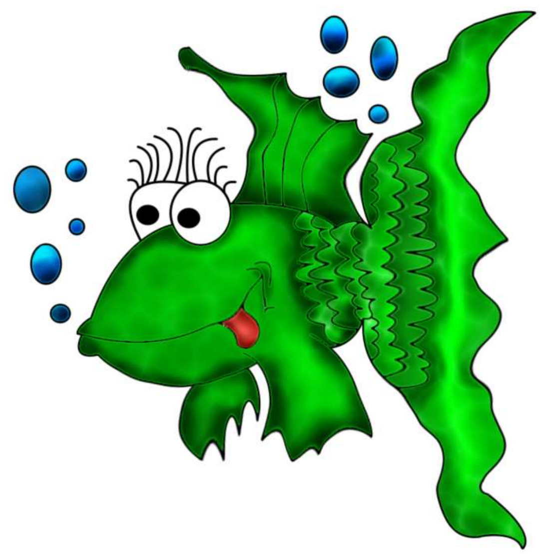


WHAT IS THE AREA OF THE OVERLAPPING REGION ?

2009 REGIONAL CHALLENGE

PROBLEM TWO



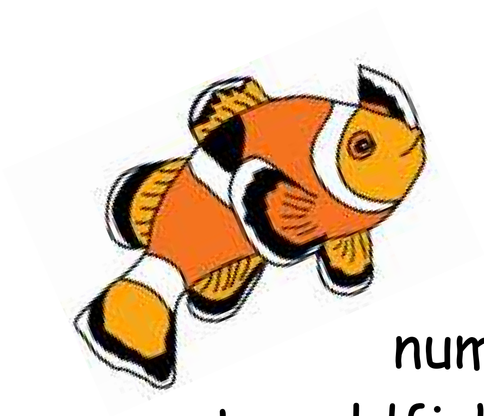


Freddie Fish had his aquarium tilted on a 45° angle. The water made a line from A to B. Point A is the midpoint of the side WX and point B is midpoint of the side XY. Each side of Freddie's tank is 40 cm.

WHEN THE TANK WAS RETURNED TO LEVEL, HOW HIGH WAS THE WATER IN FREDDIE'S TANK?

2009 REGIONAL CHALLENGE

PROBLEM THREE

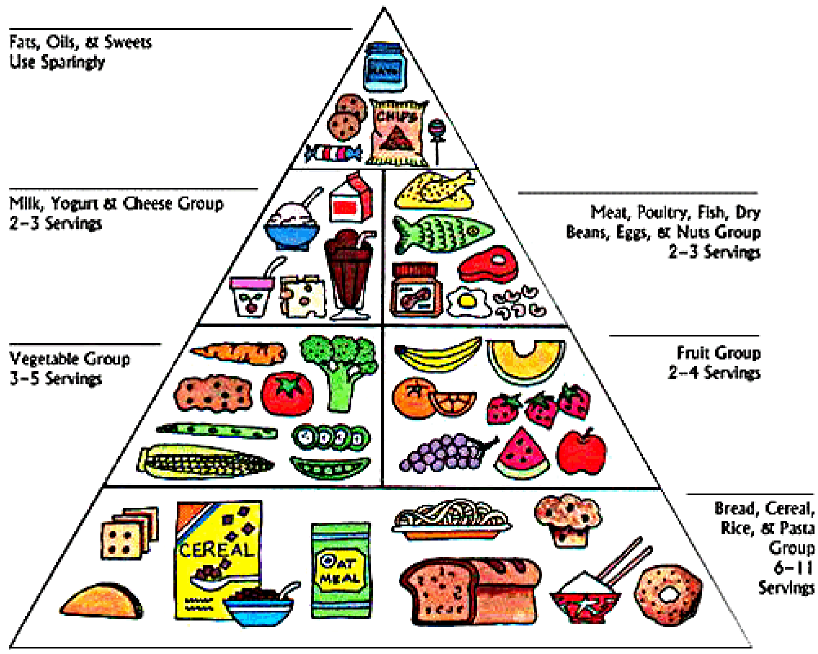


Jane has two aquariums. In one aquarium, the ratio of the number of clownfish to goldfish is 2:3. In the other, the ratio of clownfish to goldfish is 3:5.

IF JANE HAS 20 CLOWNFISH IN TOTAL, WHAT IS THE LEAST NUMBER OF GOLDFISH THAT SHE COULD HAVE?

2009 REGIONAL CHALLENGE

PROBLEM FOUR



Ms. Pleasant was discussing healthy eating with her class of 28 students and gave a survey to investigate what her students ate the night before. After getting the survey back the results were:

12 students ate vegetables
15 students ate grains
10 students ate meat
3 students ate vegetables and grains
2 students ate vegetables and meat
4 students ate grains and meat
1 student ate vegetables, grains and meat

HOW MANY STUDENTS DID NOT EAT VEGETABLES, GRAINS OR MEAT?

2009 REGIONAL CHALLENGE

PROBLEM FIVE

A plane carrying 8 celebrities starts out travelling 300km per hour. After an hour passes, one celebrity reaches their destination and parachutes from the plane, increasing the rate of speed by an additional 50km/hr. This increase in speed occurs once every hour until there are no celebrities left. When the final celebrity jumps, the plane gains the last 50km/ hr, and maintains that speed when it turns around. It takes a short-cut back to the original destination, saving 300km of travel.

HOW MANY HOURS WILL IT TAKE FOR THE PLANE TO RETURN TO ITS ORIGINAL STARTING PLACE?

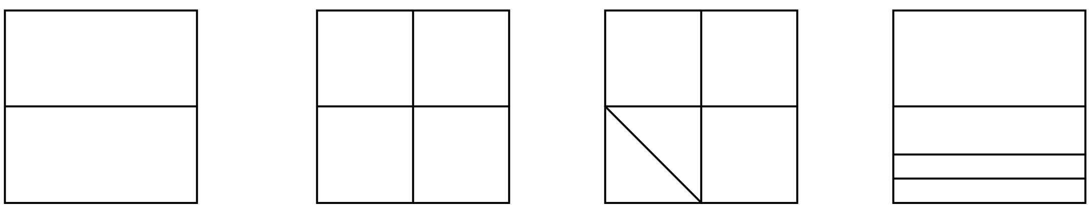
Solutions REGIONAL CHALLENGE
PROBLEM ONE

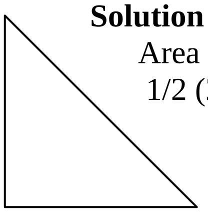
NOT AVAILABLE

Solutions REGIONAL CHALLENGE
PROBLEM TWO

Freddie fish geometry

Solution 1 Recognize properties of squares. Isolate triangle that is 1/2 of 1/2 of 1/2 or the tank. See below. 1/8 of the tank tipped so the base is horizontal is 1/8 of 40 cm or 5 cm.





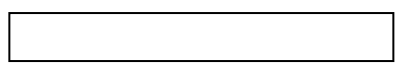
Solution 2 Isolate triangle

Area of triangle = area of rectangle

1/2 (20 cm) (20 cm) = 40 cm x h

$200\ cm^2 = 40$ cm x h

5 cm = h



Solution 3 Calculate volume of water.
The volume of water remains the same, so

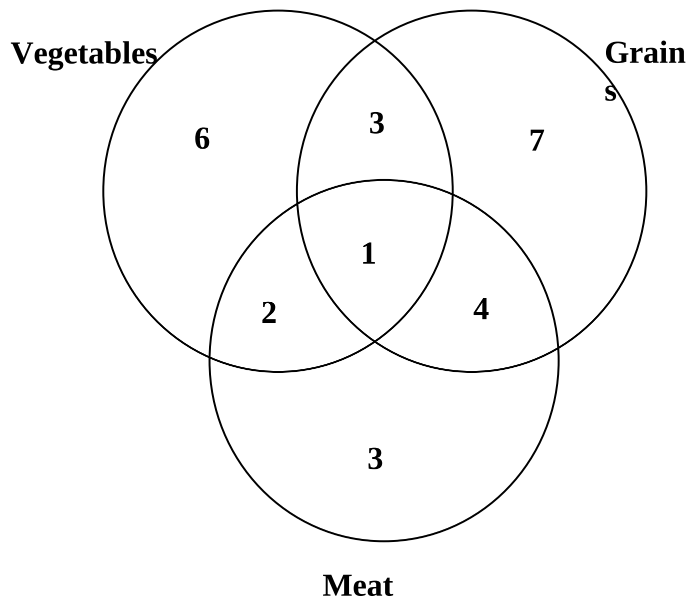
Triangular based prism = square base column
1/2 (20 cm)(20 cm)(40 cm) = h x (40 cm)(40 cm)
$8000\ cm^3$ = h x $1600\ cm^2$
5 cm = h

Solutions REGIONAL CHALLENGE PROBLEM THREE

NOT AVAILABLE

Solutions REGIONAL CHALLENGE PROBLEM FOUR

1) Use a Venn Diagram:



Then, add up the numbers to get 26, which shows 2 students (28 – 26 = 2) went hungry.

Calculate the number of students falling into each possible combination:

Just Vegetables: 12 - [(4 + 3) – 1] = 6
Just Grains: 15 - [(4 + 5) – 1] = 7
Just Meat: 10 = [(3 + 5) – 1] = 3
Vegetables & Grains: 4 – 1 = 3
Vegetables & Meat: 3 – 1 = 2
Grains & Meat: 5 – 1 = 4
Vegetables, Grains and Meat: 1

Where the subtractions above are to eliminate double counting in the previous term. Then, add the values to get 26, which shows 2 students (28 – 26 = 2) went hungry.

Add all of the given values:

12 + 15 + 10 + 4 + 3 + 5 + 1 = 40

Add the values that represented 2 categories (were double counted):

4 + 3 + 5 = 12

Add the values that represented 3 categories (were triple counted):

1

Then, correct the original total of 40:

40 – 12 – (2 x 1) = 26

Which shows 2 students (28 – 26 = 2) went hungry.

Solutions REGIONAL CHALLENGE PROBLEM FIVE

Solution Paths:

1- Write out a list (or chart):
300 km for first hour,
1st celebrity jumps, 350km for second hour
400km for 3rd hr
450km for 4th hr
500km for 5th hr
560km for 6th hr
600km for 7th hr
650km for 8th hr
When the eighth celebrity jumps out, 700km per hour is the final speed to be travelled on the return flight

To find the km traveled in the 8 hrs, add all numbers in the list from 300km to 650km = 3800km

Subtract 300km for the short cut (3800km-300km) = 3500 km to travel on the return flight.

At 700km per hour, this will take (3500km)/(700km/hr)=5 hours

2- Original 300 km x 8 hrs= 2400; Then, we have to account for the 50km/hr added each hour. The first 50km will be added for 7 hrs, the second for 6, the third for 5 etc. So (7 + 6 + 5 + 4 + 3 +2 + 1= 28 times that 50 must be added). Multiply 28 by 50 = 1400km. Add this to 2400= 3800 total km traveled. Subtract the 300 km saved from the short cut= 3500. Km
This has to be divided by the final speed attained. To find this, 300 + (50x8) = 300 + 400 = 700km/hr. 3500 km / 700 km/hr = 5 hrs

3- We know that the original flight takes 8 hours (because 8 celebrities to drop off). On the return flight, the plane travels faster, saving a certain percentage of each hour.
300km travelled for the first original hour can be removed, as 300 km is saved from the short cut. The remaining time spent to travel the remaining km (over the other 7 hours) will be shortened due to being travelled faster. They become: (note, a line through numbers means they repeat)

350km per hr/700km per hr=.5 of the original 1 hr spent travelling at the speed of 350

400km per hr/700km per hr=.571428~~571428~~ of the original 1 hr

450km per hr/700km per hr=.6428571~~428571~~ of the original 1 hr

500km per hr/700km per hr=.714285~~714285~~ of the original 1 hr

550km per hr/700km per hr=.78571~~428571~~ of the original 1 hr

600km per hr/700km per hr=.857142~~857142~~ of the original 1 hr

650km per hr/700km per hr=.928571~~428571~~ of the original 1 hr

Add these together to get 5 hrs of travel.

2009 REGIONALS CHALLENGE

ANSWER KEY

PROBLEM ONE -	25
PROBLEM TWO -	5
PROBLEM THREE -	31
PROBLEM FOUR -	2
PROBLEM FIVE -	5