# 2009 ZONE Grade 7 PROBLEM ONE 

Jannah and Erin made a total of 99 chocolate chip cookies for the grade seven bake sale. Jannah made 9 more cookies than Erin. Erin made an amount of cookies that, coincidentally, was made up of the same digits as the amount of cookies that Jannah made, but with the order of the digits reversed.


HOW MANY COOKIES DID
JANNAH MAKE FOR THE BAKE SALE?

# 2009 ZONE Grade 7 PROBLEM TWO 



Frances is a huge fan of the famous mathematician Fibonacci. To honor him and her favourite number, three, she invented the Tribonacci Sequence.
In a "Tribonacci
Sequence", each number after the third number is the sum of the preceding three numbers. For example: $1,2,3,6,11,20$,...

NOTE THAT $1+2+3=6$, THEN $2+3+6=11$, THEN 3 $+6+11=20 \ldots$ ETC.

## IF THE FIRST FIVE NUMBERS OF A TRIBONACCI SEQUENCE ARE P, Q, 86, 158, AND 291,

## WHAT IS THE VALUE OF P?

# 2009 ZONE Grade 7 PROBLEM THREE 

Jen and her five friends are protesting the outcome of their school's bowling competition. Their team's average was posted as 89. (NOTE: There are 6 team members in total) However, when Jen noted that her score of 86 had been mistakenly recorded as a 68, she was angry because they had just missed winning first place! The judges have agreed to recalculate the results.


## 2009 ZONE Grade 7 PROBLEM FOUR

Craig has 4 more brothers than sisters.


HOW MANY MORE BROTHERS
THAN SISTERS DOES
CRAIG'S SISTER ERIN HAVE?
(HINT: TEST THE SIMPLEST SCENARIO)

# 2009 ZONE Grade 7 PROBLEM FIVE 



Josh and Selena are learning how to figure skate. At practice, every three minutes Josh does a triple sow-cow and every four minutes Selena does a spiral. They have both practiced from 2-4pm.

DOES SELENA NEED TO PRACTICE HER SPIRAL IN ORDER TO CATCH UP TO THE NUMBER OF TIMES THAT JOSH PRACTICED HIS TRIPLE SOW COW?

## Solutions ZONE GRADE SEVEN

## PROBLEM ONE

## Method 1: Logical Reasoning

Let $A B$ and $B A$ represent the respective number of cookies purchased by Jannah and

$$
A B+\quad A B-
$$

Erin. The conditions given in the question are listed to the right. [1] $\frac{B A}{99}[2] \frac{B A}{9}$.
From the first condition it can be observed that the sum of $A$ and $B$ is 9 . From the tens column of the second condition it can be observed that $B$ is one less than $A$. Therefore, $A$ and $B$ are consecutive numbers with a sum of 9 . Given $A B$ must be larger than $B A, A$ must be larger than $B$. $A$ and $B$ must be 5 and 4 respectively. Therefore, Jannah purchased 54 cookies from the bake sale.

## Method 2: Trial and Error Based on Sum and Difference Constraints

It is possible to determine that Jannah purchased 54 cookies by trial and error in 2 ways.

1. Find the difference of any two numbers whose sum is 99 . The only set of these numbers whose difference is 9 will be 54 and 45 . Since Jannah purchased more cookies than Erin, Jannah purchased 54 cookies from the bake sale.
2. Find the sum of any two numbers whose difference is 9 . The only set of these numbers whose sum is 99 will be 54 and 45 . Since Jannah purchased more cookies than Erin, Jannah purchased 54 cookies from the bake sale.

## Method 3: Make a Table of Values Based on Reversed Digits Constraint

 Due to the fact that the number of cookies purchased by Jannah and Erin combined is 99 and the fact that Erin purchased a number of cookies made up of the same digits as Jannah's number of cookies but with the order reversed, we know that the sum of the digits of the number of cookies Jannah purchased must be 9. A table can be constructed of all of the possible numbers of cookies purchased by Jannah, the resulting number of cookies purchased by Erin and the difference between them. The numbers whose difference is 9 will be the number of cookies purchased by Jannah and Erin.| Number of Cookies <br> Jannah bought | 81 | 72 | 63 | 54 |
| :---: | :---: | :---: | :---: | :---: |
| Number of Cookies <br> Erin bought | 18 | 27 | 36 | 45 |
| Difference | $\mathbf{6 3}$ | $\mathbf{4 5}$ | $\mathbf{2 7}$ | $\mathbf{9}$ |

Therefore, Jannah purchased 54 cookies from the bake sale.

# Solutions ZONE GRADE SEVEN <br> PROBLEM TWO 

NOT AVAILABLE
25

# Solutions ZONE GRADE SEVEN <br> PROBLEM THREE 

NOT AVAILABLE
92

## Solutions ZONE GRADE SEVEN PROBLEM FOUR

## Method 1

Consider the total number of siblings. The total number of siblings is Craig + 4 + sisters. Then the family has 5 more brothers than sisters. Erin has 1 less sister than the total number of sisters because Erin was already counted as one sister. Therefore Erin has 5+1=6 more brothers than sisters.

## Method 2

Assume that Craig has various numbers of sisters, then find a pattern. Suppose that Craig has 2, 3, 4, or 5 sisters. Then he has $6,7,8$, or 9 brothers. Erin therefore has $1,2,3$, or 4 sisters and $7,8,9$, or 10 brothers. In all cases, Erin has 6 more brothers than sisters. It seems reasonable that this last statement would be true regardless of the number of sisters assigned to Craig.

## Method 3

Use algebra. Let $M=$ number of brothers and let $F=$ number of sisters. There are $5+F$ males, and Erin has $F-1$ sisters. We subtract $5+F-(F-1)=6$. Erin has 6 more brothers than sisters.

## Solutions ZONE GRADE SEVEN <br> PROBLEM FIVE

1. Josh $=3 \mathrm{~min}$
Josie $=4 \mathrm{~min}$
Josh : $120 \mathrm{~min} / 3 \mathrm{~min}=40$ times
Josie : $120 \mathrm{~min} / 4 \mathrm{~min}=30$ times
$40-30=10$ times
10 times $\times 4$ minutes $=40$ minutes
2. Josh: 1 jump $/ 3 \mathrm{~min} \rightarrow 60 \mathrm{~min} / 3=20$ jumps/hour 2 hours $=40$ jumps
Josie: 1 jump/ $4 \mathrm{~min} \rightarrow 60 \mathrm{~min} / 3=15 \mathrm{jumps} /$ hour 2 hours $=30$ jumps
$40-30=10$ times
10 times $\times 4$ minutes $=40$ minutes
3. Josh: 1 jump $/ 3 \mathrm{~min} \rightarrow$ slope $=1 / 3$

Josie: 1 jump $/ 4 \min \rightarrow$ slope $=\frac{1}{4}$


The chart shows that Josh has practiced 40 times and Josie has practiced 30 times. Josie needs to practice 10 more times to catch up to Josh.

10 times $\times 4$ minutes $=40$ minutes

## 2009 ZONES <br> GRADE SEVEN

ANSWER KEY

PROBLEM ONE - 54

PROBLEM TWO - 25

PROBLEM THREE - 92

PROBLEM FOUR - 6

PROBLEM FIVE - 40

## 2009 ZONES <br> GRADE SEVEN MATH OFF PROBLEM

Sean let Carol play with his checkers set which included 60 red and 60 black checkers. When he got the set back Sean stacked the checkers in an equal number of piles that were either all red chips or all black chips. Alarmingly, he noticed that Carol had somehow lost 24 of his red checkers!

## WHAT IS THE GREATEST NUMBER OF CHECKERS

 THAT EACH OF HIS PILES CAN HAVE?

## 2009 MATH OFF PROBLEM GRADE SEVEN <br> ANSWER - 12

## 2009 MATH OFF PROBLEM GRADE SEVEN

Solution 1: Finding Factors
Factors of 60: 1,2,3,4,5,6,10,12,15,20,30,60

## Factors of 36:

1,2,3,4,6,9,12,18,36
Since 12 is the highest common factor, that is the maximum number of chips in each pile for them to be equal.
Solution 2: There are only 36 red chips (60-24), and 60 black chips. The fewer number of piles Sean has, the more checkers there will be in each pile. Make a table starting with one pile of each, and once you have the same number of checkers in the second and forth column, with no checkers left over, we know we have the greatest number of checkers that each pile can have.

| Number of Piles | Number of Red <br> Checkers | Number of Red <br> Checkers left over | Number of Black <br> Checkers | Number of Black <br> Checkers left over |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 36 | 0 | 60 | 0 |
| 2 | 18 | 0 | 30 | 0 |
| 3 | 12 | 0 | 20 | 0 |
| 4 | 9 | 0 | 15 | 0 |
| 5 | 7 | 1 | 12 | 0 |

If we have 3 piles of 12 red checkers, and 5 piles of 12 black checkers, we will have 12 checkers in each of the piles, with no checkers left over.
Solution 3: Trial and Error
We know that the maximum number of red checkers in a pile would be 36 (all of them), so start a table with a pile of 36 red checkers, and keeping making them into more and more piles. Do the same with your black checkers until there are no black checkers left over.

| Number of Piles of Red Checkers | Number of Checkers in each Pile | Are there any black checkers left <br> over? |
| :--- | :--- | :--- |
| 1 | 36 | $60 / 36=5 / 3=12 / 3$ yes Or <br> $60 / 36=5 / 35$ is not divisible by 3 |
| 2 | 18 | $60 / 18=10 / 3=31 / 3$ yes Or <br> $60 / 18=10 / 31+0=1$, and 1 is not <br> divisible by 3 |
| 3 |  | $60 / 12=5$ no |

Sean would need to arrange his piles in groups of 12

## 2009 ZONE Grade 8 PROBLEM ONE

Megan was playing in the "Knock 'em Down" bowling tournament. Her first three scores were 195, 187 and 221. After her fourth game, the average of her four scores was 205.

## WHAT IS THE SUM OF MEGAN'S BEST THREE GAMES?

## 2009 ZONE Grade 8 PROBLEM TWO

The SPCA has discovered an abandoned farmhouse with homeless cats. There are seven families willing to adopt the animals. If the cats are equally divided among six families, four cats will
 still be homeless. If the cats are equally divided among five families, three cats will still be homeless. If the cats are equally divided among three families, one cat will remain homeless. When the cats are adopted by the seven families, no cat will be left homeless.

ON THE FARM, THE SPCA FOUND THE LEAST NUMBER OF CATS THAT MET ALL OF THE ABOVE CONDITIONS.

## HOW MANY CATS WILL EACH OF THE SEVEN ADOPTING FAMILIES RECEIVE?

# 2009 ZONE Grade 8 PROBLEM THREE 



A large ice cube, that you intended to carve into an amazing sculpture, had an initial volume of $216 \mathrm{~cm}^{3}$.
Unfortunately, the sun was very hot that day. By the time you were ready to begin carving, your ice cube

had melted down to $\overline{6}$ of its original surface area, while still maintaining its original cubic shape.

# WHAT IS THE DIFFERENCE IN SURFACE AREA BETWEEN THE ORIGINAL ICE CUBE AND THE MELTED ONE? 

# 2009 ZONE Grade 8 PROBLEM FOUR 

Peter's gumball machine is half full. After onehalf of a kilogram of gumballs is added, the gumball machine is twothirds full.


HOW MANY<br>KILOGRAMS OF GUMBALLS DOES<br>THE GUMBALL<br>MACHINE HOLD<br>WHEN IT IS FULL?

## 2009 ZONE Grade 8 PROBLEM FIVE

Eric is planning a pirate treasure hunt. There will be 10 people per team.

There's so much to do before the party that Eric doesn't have time to make the eye patches for his guests. After buying the materials for the patches Eric has $\$ 75$ remaining in the eye patch budget and he wants to spend it all. His nephew can make 5 eye patches in 36 minutes.

WHAT IS THE HOURLY WAGE, IN PENNIES, THAT ERIC WILL PAY HIS NEPHEW TO MAKE ENOUGH EYEPATCHES FOR 25 TEAMS?

## Solutions ZONE GRADE EIGHT PROBLEM ONE

Method 1 If the average of the four scores must be 205 , then the sum is 4 $\times 205=820$. The sum of the three given scores is $195+187+221=603$. The fourth score is equal to $820-603=217$. The sum of the best three scores is 633 .
Method 2 The first score is 10 below the average; the second score is 18 below the average; and the third score is 16 over the average. Thus, the sum of the three scores is 12 below the three average scores. Thus, the fourth score must be 12 above the average score, which is $205+12=217$.

## Method 3

Using algebra, let $S$ represent the fourth score.
Definition of Average
Multiply both sides of (1) by 4
Simplify (2)
Subtract 603 from both sides of (3)
(1) $(195+187+221+S) \div 4=205$
(2) $195+187+221+S=820$
(3) $603+S=820$ $S=820$

- 603

Therefore the fourth score is 217 $S=217$
Her best three games remain consistent for all three solution paths $\rightarrow 221+$ $217+195=633$.

## Solutions ZONE GRADE EIGHT

## PROBLEM TWO Answer: 4 Solution Methods:

One solution path is for students to write down numbers that are a multiple of $6(+4)$ (i.e. $10,16,22,28,34 \ldots$ ), numbers that are a multiple of $5(+3)$ (i.e. $8,13,18,23,28,33 \ldots$ ), numbers that are a multiple of $3(+1)$ (i.e. $4,7,10,13,16 \ldots$ ) and then see which number is the first one they encounter common to all of these scenarios. This would be 28. Dividing 28 into 7 parts, students would determine that the each family would receive 4 cats.

A second method is to find the answer through trial and error. For example, in the first statement, if each person had one cat, the total would be 10 . However, that does not satisfy the other statements. The student would then try 2,3 , etc. The first answer that would satisfy all statements would be a total of 28 cats (meaning that each person would get 4 cats when they are divided among 7 people).

A third option would be to rule out that the answer is a multiple of 6 , 5 or 3 . We know this because when the cats were divided among that number of families, there was a remainder. So using multiples of 2,4 , and 7,8 , and 9 , we find that 28 is the first number that satisfies the given conditions.

## Solutions ZONE GRADE EIGHT PROBLEM THREE

## Solution 1:

Volume of cube $=X^{3}$
The initial volume of the ice cube is $216 \mathrm{~cm}^{3}$ so the length of each side must be $X=\sqrt[3]{216}=6 \mathrm{~cm}$.
Surface area of a cube $=6(X)^{2}$
[where $X$ is the length of the side]
Surface area $=6(6)^{2}=216 \mathrm{~cm}^{2}$
After five minutes: $\frac{1}{6}$ of surface area
$216 \times \frac{1}{6}=36 \mathrm{~cm}^{2}$
Difference in Surface Area is 216-36 = 180

## Solution 2:

Volume of cube $=X^{3}$
The initial volume of the ice cube is $216 \mathrm{~cm}^{3}$ so the length of each side must be $X=\sqrt[3]{216}=6 \mathrm{~cm}$.
Initial surface Area $=6(X)^{2} \quad$ New Surface area $=\frac{1}{6}\left(6(X)^{2}\right)$
New Surface area $=\frac{1}{6}\left(6(6)^{2}\right)$ New Surface area $=36 \mathrm{~cm}^{2}$

Solution 3:
(Volume of a Cube $=$ Side $^{3}$ )
Therefore Side $=\sqrt[3]{\text { volume }}$
(Surface Area of a Cube $=6\left(\right.$ Side $^{2}$ )
Therefore Side $=\sqrt[2]{\text { Surfacearea } / 6}$
So $\sqrt[3]{\text { volume }}=\sqrt[2]{\text { Surfacearea/6 }}$
Original cube:
$\sqrt[3]{216 \mathrm{~cm} 3}=\sqrt[2]{\text { Surfacearea } / 6}$
$6 \mathrm{~cm}=\sqrt[2]{\text { Surfacearea } / 6}$
Squaring both Sides:
$36 \mathrm{~cm}^{2}=$ Surface area / 6
And Surface Area $=6\left(36 \mathrm{~cm}^{2}\right)=216 \mathrm{~cm}^{2}$
New cube:
New Surface area $=\frac{1}{6}\left(216 \mathrm{~cm}^{2}\right)=36 \mathrm{~cm}^{2}$
Difference $=180$

## Solutions ZONE GRADE EIGHT

## Method \#1:

$X=$ total $\#$ of tons the gumball machine can hold.
$\frac{1}{2}$
$\overline{2} \mathrm{x}+\frac{1}{2} \mathrm{~kg}=\frac{2}{3} \mathrm{X}$
$\frac{1}{2} \mathrm{~kg}=\frac{2}{3} \mathrm{X}-\frac{1}{2} \mathrm{X}$
$\frac{1}{2} \mathrm{~kg}=\frac{1}{6} \mathrm{X}$
$\mathrm{x}=3 \mathrm{~kg}$
Method \#2
Have $1 / 2=1.5 / 3$.
And you know that you need 0.5 to get to $2 / 3$ full.
Thus 0.5 T means that the machine is $0.5 / 3$ full, or $0.1 \overline{6}$ full.
Since full is $1,1 \div 0.1 \overline{6}=6 \quad$ And $6 \times 0.5=3$
Solutions ZONE GRADE EIGHT PROBLEM FIVE

## Method A

$\frac{250 \text { patches }}{x \mathrm{~min}}=\frac{5 \text { patches }}{36 \mathrm{~min}}$
(36) $\frac{250}{x}=5$
(x) $\frac{9000}{x}=5 x$
$\frac{9000}{5}=\mathrm{x}$
$x=1800$ minutes;

$$
\frac{1800 \mathrm{~min}}{60}=30 \text { hours }
$$

Hourly wage $=\frac{\$ 75}{30} ; \quad$ Hourly wage $=250$ pennies
Eric would have to pay his nephew an hourly wage of 250 pennies to make the patches.
Method B:
$\frac{25 \text { ships }}{x \min }=\frac{1 \text { ship }}{72 \mathrm{~min}}$
(72) $\frac{25}{x}=1$
(x) $\frac{1800}{x}=1 x$
$x=1800$ minutes; $\quad \frac{1800 \mathrm{~min}}{60}=30$ hours
Hourly wage $=\frac{\$ 75}{30} ; \quad$ Hourly wage $=\$ 2.50$
Eric would have to pay his nephew an hourly wage of 250 pennies to make the patches.

## Method C:

It takes 36 minutes to make 5 patches; therefore it will take 72 minutes to make 10 patches, or enough patches per one ship:
25 ships x $72 \mathrm{~min} / \mathrm{ship}=1800$ minutes to make enough patches for 25 ships.
$\frac{1800 \mathrm{~min}}{60}=30$ hours

Hourly wage $=\frac{\$ 75}{30} ; \quad$ Hourly wage $=\$ 2.50$

Eric would have to pay his nephew an hourly wage of 250 pennies to make the patches.

## 2009 MATH OFF PROBLEM <br> GRADE EIGHT <br> ANSWER - 20

2009 ZONES
GRADE EIGHT
ANSWER KEY

## PROBLEM ONE - 633

PROBLEM TWO - 4

PROBLEM THREE - 180

PROBLEM FOUR - 3

PROBLEM FIVE - 250

## 2009 ZONES

## GRADE EIGHT

MATH OFF PROBLEM


Brian loves bananas. In
fact, he loves them so
much that he decided to only eat bananas for five days. He started his banana diet on a Monday.
Every day he ate 6 more bananas than the previous day. By the end of the day on Friday, Brian had consumed 100 bananas!

BRIAN EAT ON WEDNESDAY?

## 2009 ZONE Grade 9 PROBLEM ONE



Sunnybrook Junior High had a snow day on Friday. Megan spent the afternoon building a snowman that was 144 cm tall. On Saturday, Megan re-measured her snowman and discovered that it had melted a bit and was now only 89 cm tall. Megan measured the height of her snowman each morning after that and recorded her data in a table:

| Day |  |  | of | the |
| :---: | :---: | :---: | :---: | :---: |
| Height of <br> Snowman | 144 | Friday | Saturday | Sunday |
| 89 | 55 | Monday | Tuesday |  |

Assuming that the snowman continues to melt following the pattern in Megan's table, on what day of the week will the snowman disappear completely?


## 2009 ZONE Grade 9 PROBLEM TWO

Three hockey sticks and two pucks weigh 32 kg . Four hockey sticks and three pucks weigh 44 kg . All the sticks weigh the same and all the pucks weigh the same.


# WHAT IS THE WEIGHT OF TWO HOCKEY STICKS AND ONE PUCK? 

# 2009 ZONE Grade 9 PROBLEM THREE 



Prince Anthony has built a large square pool inside of his circular courtyard. The corners of his pool go to the edge of the courtyard as shown in the diagram.

Prince Anthony has a dog named Cocoa. Cocoa loves to swim and frequently escapes from the castle to play in the pool. The prince would like to make a plastic cover for the pool so Cocoa can only get into the pool when a human is present. The circumference of the courtyard is 37.68 metres. Plastic covering costs 100 dollars per square metre.


## How much will the cover for the square pool cost Anthony?

Use $\Pi=3.14$
Circumference $=2 \pi r$

## 2009 ZONE Grade 9 PROBLEM FOUR



Brittany has placed the three balls shown above in a bag. She draws one ball from the bag, then depending on the number on the ball, eats that many marshmallows and then returns the ball to the bag.
She repeats this process two more times.

## WHAT IS THE PROBABILITY <br> THAT BRITTANY HAS EATEN LESS THAN 8 MARSHMALLOWS?


(YOUR ANSWER MUST BE A FRACTION IN SIMPLEST FORM!!!)

# 2009 ZONE Grade 9 PROBLEM FIVE 

Mr. MacAnswer has created a special math quiz for his students. It is made up of 20 tricky math questions. Each correct answer earns 5 points. Each incorrect answer reduces the score by 2 points and unanswered questions score 0 points. Adam has a total of 59 points.

## HOW MANY <br> QUESTIONS <br> DID ADAM <br> OMIT?



# Solutions ZONE GRADE NINE PROBLEM ONE <br> NOT AVAILABLE <br> Answer: Wednesday <br> Solutions ZONE GRADE NINE <br> PROBLEM TWO 

Answer: 20
Solutions ZONE GRADE NINE

## PROBLEM THREE

Method 1: Find individual areas of triangles I, II, III, and IV
Solve for Diameter:
Circumference $=2 \Pi r=\Pi d$
$37.68 \mathrm{~m}=3.14 \mathrm{~d}$
$\mathrm{d}=12 \mathrm{~m}$
Find Area of Square:
Triangles I, II, III, and IV are all congruent and have equal areas.
Since segments AC and BD are diameters of length 12 m , the base and height of each
triangle is 6 m .
Area of triangle $\mathrm{I}=\frac{\text { BaseXHeight }}{2}=\frac{6 X 6}{2}=\frac{36}{2}=18$. The area of triangle I is $18 \mathrm{~m}^{2}$. Since triangles I, II, III, and IV all have the same area, square ABCD has an area of $72 \mathrm{~m}^{2}$.
Find cost of pool:
$72 m^{2} x \frac{\$ 100}{m^{2}}=\$ 7200$
The cover for the square pool will cost Prince Anthony $\$ 7200$.
Method 2: Rearrange triangles into 2 squares
Solve for Diameter:
Circumference $=2 \Pi r=\Pi d$
$37.68 \mathrm{~m}=3.14 \mathrm{~d}$
$\mathrm{d}=12 \mathrm{~m}$
Find Area of Square:
Since the diameter is 12 m , the base and height of each triangle is 6 m . The triangles can be arranged by aligning the hypotenuse of each triangle to the
 hypotenuse of another triangle. The result is 2 squares with bases and heights of 6 m as shown in the diagram below. The area of each square is $36 \mathrm{~m}^{2}$ therefore the area of square ABCD is $72 \mathrm{~m}^{2}$.
Find cost of pool:
$72 m^{2} x \frac{\$ 100}{m^{2}}=\$ 7200$
The cover for the square pool will cost Prince Anthony $\$ 7200$.
Method 3: Rearrange the triangles into 1 rectangle
Solve for Diameter:
Circumference $=2 \Pi r=\Pi d$
$37.68 \mathrm{~m}=3.14 \mathrm{~d}$
$\mathrm{d}=12 \mathrm{~m}$
Find Area of Square:
Since the diameter is 12 m , the base and height of each triangle is 6 m . The triangles can be arranged by aligning the hypotenuse of each triangle to the hypotenuse of another triangle to form two squares. The squares can be arranged such that the two square are touching along one side. The result is 1 rectangle of height 6 m and base 12 m as shown in the diagram below.


The area of the rectangle is $72 \mathrm{~m}^{2}$ therefore the area of square $A B C D$ is $72 \mathrm{~m}^{2}$.
Find cost of pool:
$72 m^{2} x \frac{\$ 100}{m^{2}}=\$ 7200$
The cover for the square pool will cost Prince Anthony $\$ 7200$.

## Solutions ZONE GRADE NINE

Answer: 23

## 2009 MATH OFF PROBLEM

2009 ZONES
GRADE NINE ANSWER KEY
PROBLEM ONE - Wednesday

## PROBLEM TWO - 20

PROBLEM THREE - 7200

PROBLEM FOUR - $\frac{23}{27}$

PROBLEM FIVE - 4

## 2009 ZONES GRADE NINE MATH OFF PROBLEM

The rectangular table holding the guestbook for Jane's birthday party is 3 times as long as it is wide. It is too long to fit along the wall by the door where the guests would be entering. It would fit if it were a square. If it were 3 meters shorter and 3 meters wider, it would be a square.


# HOW LONG IS THE LONGEST SIDE OF THE RECTANGULAR TABLE? 

## 2009 ZONE CHALLENGE PROBLEM ONE

Captain Canoehead is growing a batch of plankton that will re-populate his lake.
The amount of plankton he is growing doubles in number every day. It takes sixteen days to grow a
 large enough batch to save the lake.


## HOW MANY DAYS DOES IT TAKE CAPTAIN

CANOEHEAD TO GROW ONE QUARTER OF THE AMOUNT OF PLANKTON THAT HE NEEDS TO SAVE THE LAKE?

## 2009 ZONE CHALLENGE PROBLEM TWO

Fruit Fluffs Cereal company is looking to make their boxes taller. Donald Fluff decided to increase the height by $30 \%$, and reduce the width so that the new box will hold the same amount of cereal as the old one but give the
 impression that it is larger. The width of the original box is 30 cm .

## WHAT IS THE WIDTH OF THE NEW BOX?

## NOTE: ROUND YOUR ANSWER TO

 THE NEAREST WHOLE NUMBER!!!
# 2009 ZONE CHALLENGE PROBLEM THREE 

An F-14 Tomcat fighter jet can travel 10000 metres in 50 seconds.


## AT THIS SPEED, HOW MANY KILOMETRES CAN IT TRAVEL IN ONE HOUR?

## 2009 ZONE CHALLENGE PROBLEM FOUR

Jackie Chan treated his film crew to a Chinese lunch. Everyone in the crew had one pork bun, one shrimp dumpling and one sticky rice bundle. Pork buns are served two to a plate, shrimp dumplings are served three to a plate, and sticky rice bundles are served two to a plate. When the waiter came to add up the plates and tally the bill, there were 48 plates and no leftovers.


## How many people are in Jackie's

 film crew?
# 2009 ZONE CHALLENGE PROBLEM FIVE 

A family is weaning their kitten from squishy soft food to crunchy hard food over 10 days. Each day the kitten eats two cups of food in total. The first day, the kitten eats $10 \%$ hard food and $90 \%$ soft food. The second day, the kitten eats 20\% hard food and $80 \%$ soft food. The third day, the kitten eats $30 \%$ hard food and $70 \%$ soft food. This pattern of increasing hard food and decreasing soft food continues until the kitten is eating only hard food.


## Solutions ZONE CHALLENGE <br> PROBLEM ONE

## 1. Captain Canoehead

Solution 1 Make a chart assigning day 1 with 1 unit of plankton and doubling the amount with each passing day ending with day 16 .

| day | portion |
| :--- | :--- |
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 8 |
| 5 | 16 |
| 6 | 32 |
| 7 | 64 |
| 8 | 128 |
| 9 | 256 |
| 10 | 512 |
| 11 | 1024 |
| 12 | 2048 |
| 13 | 4096 |
| 14 | 8192 |
| 15 | 16384 |
| 16 | 32768 |

$1 / 4$ of total $=1 / 4$ of $32768=8192$
This number corresponds to 2 days ago or day 14
Solution 2 work backwards dividing by two
day 16 total amount
day $15 \quad 1 / 2$ of total
day $14 \quad 1 / 4$ of total
Solution 3 use algebra
$(1 / 2)^{n}=1 / 4$
$\mathrm{n}=2$
$16-2=14$
2 days ago was Day 14

## Solutions ZONE CHALLENGE <br> PROBLEM TWO

Solution Paths to Question Number Three: Answer $=23 \mathrm{~cm}$

1) volume $=$ length $\times$ width $\times$ height initial volume $=20 \mathrm{~cm} \times 30 \mathrm{~cm} \times 40 \mathrm{~cm}=24000 \mathrm{~cm}^{3}$

Added height $=0.3 \times 40 \mathrm{~cm}=12 \mathrm{~cm}$
New height $=40 \mathrm{~cm}+12 \mathrm{~cm}=52 \mathrm{~cm}$
New box volume $=$ old box volume $=24000 \mathrm{~cm}^{3}=52 \mathrm{~cm} \times 20 \mathrm{~cm} \times$ new w
$24000 \mathrm{~cm}^{3}=1040 \mathrm{~cm}^{2} \times$ (new w)
$24000 \mathrm{~cm}^{3} / 1040 \mathrm{~cm}^{2}=$ new $w$
$23 \mathrm{~cm}=$ new $w$
ANSWER $=23 \mathrm{~cm}$
2) length ${ }_{1} \times$ width $_{1} \times$ height $_{1}=$ length ${ }_{2} \times$ width $_{2} \times$ height $_{2}$
$(20 \mathrm{~cm})(30 \mathrm{~cm})(40 \mathrm{~cm})=(20 \mathrm{~cm})(1.3 \times 40 \mathrm{~cm})\left(w_{2}\right)$
$(20 \mathrm{~cm})(30 \mathrm{~cm})(40 \mathrm{~cm})=30 \mathrm{~cm} / 1.3=w_{2}=23 \mathrm{~cm} \quad$ ANSWER $=23 \mathrm{~cm}$ $(20 \mathrm{~cm})(1.3 \times 40 \mathrm{~cm})$
3) old volume $=20 \mathrm{~cm} \times 30 \mathrm{~cm} \times 40 \mathrm{~cm}=24000 \mathrm{~cm}^{3}=$ new volume
$24000 \mathrm{~cm}^{3}=[(40 \mathrm{~cm} / 10 \times 3)+40 \mathrm{~cm}] \times$ width $\times 20 \mathrm{~cm}$

$$
24000 \mathrm{~cm}^{3}=\text { width }=23 . \mathrm{cm}
$$

ANSWER $=23 \mathrm{~cm}$
$[(40 \mathrm{~cm} / 10 \times 3)+40 \mathrm{~cm}] \times 20 \mathrm{~cm}$

## Solutions ZONE CHALLENGE <br> PROBLEM THREE

## Method 1

First, 10km requires 50 seconds. Since $1 \mathrm{hr}=60$ minutes $=3600$ seconds, the jet travels 10 km for every 50 second segment of the 3600 seconds. $3600 \times \frac{10}{50}=36 \times \frac{1000}{50}=720 \mathrm{~km}$

## Method 2

10 km requires 50 seconds $=\frac{50}{60}$ minutes $=\frac{5}{6}$ minutes; 60 km require $6 \times \frac{5}{6}$ minutes $=5$ minutes. For every 5 minute segment that there is in 1 hour, the fighter jet travels 60 km . Since 1 hour divided by 5 minutes is 12 , the jet will travel $12 \times 60 \mathrm{~km}$ or 720 km in 1 hour.

## Method 3

Make a table in which 50 seconds $=\frac{5}{6}$ minutes.

| Time (minutes) | 0 | 0.8 | 1.7 | 2.5 | 3.3 | 4.2 | 5 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance $(\mathrm{km} \times 10)$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 |

Since the jet travels 60 km in 5 minutes, then it will travel $12 \times 60 \mathrm{~km}$ or 720 km in $12 \times 5$ minutes or 1 hour.

## Solutions ZONE CHALLENGE PROBLEM FOUR

## Jackie Chan

Solution 1 Recognize that the number of people must be a multiple of 6 because of the denominators and because there are no leftovers.
multiples of 3 for the $1 / 3$ plates, multiple of 2 for the $1 / 2$ plates. Every 6
people generate 8 plates. $6(1 / 2+1 / 3+1 / 2)=3+2+3=8$

| people | plates |
| :--- | :--- |
| 6 | 8 |
| 12 | 16 |
| 18 | 24 |
| 24 | 32 |
| 30 | 40 |
| 36 | 48 |

Using the chart, production of 48 plates means that there were 36 people.
Solution 2 Determine the ratio and solve for the unknown.
$1 / 2+1 / 3+1 / 2=4 / 3$
recognize that each person generates $4 / 3$ of a plate
no partial plates so the number of people must be a multiple of $2 \times 3=6$
6 people : 8 plates
? people : 48 plates
$48 / 8 \times 6=6 \times 6=36$ people
Solution 3. Solve algebraically
$x / 2+x / 3+x / 2=48 \quad x=$ number of people
$x+x / 3=48$
$4 x / 3=48$
$4 \mathrm{x}=144$
$\mathrm{x}=36$
Therefore there were 36 people.

## Solutions ZONE CHALLENGE PROBLEM FIVE

## Solution Paths:

1- Recognize that 18 cups ( 9 days) will have some soft food (as the last day will be $100 \%$ hard food). Then, either recognize that these 18 cups will be split evenly between hard and wet cat food, leaving 9 cups for each, OR add the percent of wet food each day $(10+20+30+40+50+60+70+80+90=$ 450 ) and divide that be 9 days for the average percent (50) and take 50 percent of 18 cups (9).

2- Find $90 \%$ of 2 cups for the first day $(2 \times .9=1.8), 80 \%$ of 2 cups for the second day ( $2 \times .2=1.6$ ), $70 \%$ of 2 cups for the third day $(2 \times .7=.1 .4)$ and so forth $(2 \times .6=1.2) ;(2 \times .5=1) ;(2 \times .4=.8) ;(2 \times .3=.6) ;(2 \times .2=.4) ;(2 \times .1$ -.2). Then add all of these answers together to get 9 cups in total (.1.8+1.6 $+1.4+1.2+1+.8+.6+.4+.2=9$ ). In this case, a chart may help.

3- Recognize that since $10 \%$ of 2 cups equals .2, we need to add a pattern of .0 (the day that it is 0\% hard food, 100\% soft) +.2 ( $10 \%$ hard food) + . 4 (an additional $10 \%$ hard food)+. 6 (etc) until we've reached 10 days ( $.0+.2+.4+$ $.6+.8+1+1.2+1.4+1.6$ ). For a total of 9 cups.

## 2009 ZONES <br> CHALLENGE <br> ANSWER KEY

PROBLEM ONE - 14

PROBLEM TWO - 23

PROBLEM THREE - 720

PROBLEM FOUR - 36

PROBLEM FIVE - 9

